

Lepton Flavor Mixing Pattern and Neutrino Mass Matrix after the Daya Bay Experiment

Shun Zhou *

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), D-80805 München, Germany

Abstract

The Daya Bay Collaboration has recently observed neutrino oscillations in the $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance channel, indicating that $\sin^2 \theta_{13} = 0.024 \pm 0.005$ (1σ) and $\theta_{13} = 0$ is already excluded at the 5.2σ confidence level. Now three neutrino mixing angles have been measured to a good degree of accuracy ($\theta_{12} \approx 34^\circ$, $\theta_{23} \approx 45^\circ$ and $\theta_{13} \approx 9^\circ$). Motivated by these experimental results, we propose a novel lepton flavor mixing pattern, which predicts $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{12} = (2+\sqrt{3})/(10+\sqrt{3}) \approx 0.318$ and $\sin^2 \theta_{13} = (2-\sqrt{3})/12 \approx 0.022$, together with a maximal CP-violating phase $\delta = 90^\circ$. The leptonic CP violation characterized by the Jarlskog invariant $\mathcal{J} = \sqrt{6}/72 \approx 3.4\%$ is promising to be measured in the future long-baseline neutrino oscillation experiments. Furthermore, we point out that a generalized version of μ - τ symmetry may exist in the neutrino sector and can give rise to the aforementioned mixing pattern. The possible realizations in the seesaw models with discrete flavor symmetries are also discussed.

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*E-mail: zhoush@mppmu.mpg.de

1 Introduction

Recent years have seen dramatic progress in neutrino physics [1]. Now we have been convinced by a number of elegant solar, atmospheric, accelerator and reactor neutrino experiments that neutrinos are massive and lepton flavors are mixed [2]. The lepton flavor mixing can be described by a 3×3 unitary matrix V , i.e., the Maki-Nakagawa-Sakata-Pontecorvo (MNSP) matrix [3], which is conventionally parametrized through three mixing angles (θ_{12} , θ_{23} , θ_{13}), one Dirac-type CP-violating phase δ and two Majorana-type CP-violating phases (ρ , σ). In the standard parametrization [2], the MNSP matrix reads

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P_\nu, \quad (1)$$

where $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$ and $P_\nu \equiv \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$. If neutrinos are Dirac particles, the phase matrix P_ν can be transformed away by redefining the phases of neutrino fields. From current neutrino oscillation data, one can extract the experimentally favored ranges of mixing angles (θ_{12} , θ_{23} , θ_{13}), as well as the neutrino mass-squared differences $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$, where m_i (for $i = 1, 2, 3$) are neutrino masses. Unfortunately, there are so far no constraints on the CP-violating phases. The absolute scale of neutrino masses will be determined or constrained by the future neutrinoless double beta decay experiments and cosmological observations.

One decade ago, it was recognized that the lepton flavor mixing might take the form of the so-called tri-bimaximal mixing pattern (TBM) [4]:

$$V_0 = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad (2)$$

implying $\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{13} = 0$, and vanishing CP-violating phases. Since the TBM pattern is well compatible with all the neutrino oscillation experiments, it has stimulated a torrent of model-building works based on the finite discrete flavor symmetries [5].

In the middle of last year, the T2K [6] and MINOS [7] Collaborations released their data on neutrino oscillation in the $\nu_\mu \rightarrow \nu_e$ appearance channel and found a weak hint for a nonzero θ_{13} with a significance about 2σ . The latest global-fit analysis of neutrino oscillation experiments, including the T2K and MINOS data, yields [8]

$$\sin^2 \theta_{12} = 0.312_{-0.015}^{+0.017}, \quad \sin^2 \theta_{23} = 0.52_{-0.07}^{+0.06}, \quad \sin^2 \theta_{13} = 0.013_{-0.005}^{+0.007}, \quad (3)$$

in the case of normal neutrino mass hierarchy with $\Delta m_{21}^2 = [7.59_{-0.18}^{+0.20}] \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = [2.50_{-0.16}^{+0.09}] \times 10^{-3} \text{ eV}^2$ at the 1σ confidence level. For the first time, the allowed range of the CP-violating phase is given as $\delta = [-0.61_{-0.65}^{+0.75}]\pi$. In the case of inverted neutrino mass hierarchy,

the 1σ ranges for neutrino mixing angles are

$$\sin^2 \theta_{12} = 0.312^{+0.017}_{-0.015}, \quad \sin^2 \theta_{23} = 0.52^{+0.06}_{-0.06}, \quad \sin^2 \theta_{13} = 0.016^{+0.008}_{-0.006}, \quad (4)$$

for neutrino mass-squared differences $\Delta m_{21}^2 = [7.59^{+0.20}_{-0.18}] \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = -[2.40^{+0.08}_{-0.07}] \times 10^{-3} \text{ eV}^2$, and for the CP-violating phase $\delta = [-0.41^{+0.65}_{-0.70}] \pi$. The fact that $\theta_{13} > 0$ is obtained with a significance about 3σ from the global analysis [8, 9] has led to a large number of works that attempt to explain a relatively large θ_{13} [10].

Recently the Daya Bay Collaboration has announced the observation of $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance and found the evidence for a nonzero θ_{13} with a 5.2σ significance [11]. The best-fit value together with the 1σ range is

$$\sin^2 \theta_{13} = 0.024 \pm 0.005, \quad (5)$$

which is consistent with the results from Double Chooz [12] and RENO experiments [13], and the global-fit analysis as well. The precise measurement of θ_{13} is quite important in the sense that (i) a relatively large θ_{13} makes the discovery of leptonic CP violation very promising in the future long-baseline neutrino oscillation experiments, if the CP phase itself is not highly suppressed; (ii) a relatively large θ_{13} indicates the significant deviation from the TBM pattern, and may point to a different symmetry structure of lepton flavor mixing [14].

Motivated by the new experimental results of θ_{13} , we propose a novel neutrino mixing pattern as the alternative to the TBM pattern:

$$V' = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{\sqrt{3}+1}{2\sqrt{6}} & -i\frac{\sqrt{3}-1}{2\sqrt{6}} \\ -\frac{1}{\sqrt{6}} & \frac{\sqrt{3}+1}{2\sqrt{6}} - i\frac{\sqrt{3}-1}{4} & \frac{\sqrt{3}+1}{4} - i\frac{\sqrt{3}-1}{2\sqrt{6}} \\ -\frac{1}{\sqrt{6}} & \frac{\sqrt{3}+1}{2\sqrt{6}} + i\frac{\sqrt{3}-1}{4} & -\frac{\sqrt{3}+1}{4} - i\frac{\sqrt{3}-1}{2\sqrt{6}} \end{pmatrix}. \quad (6)$$

Comparing between V' and the standard parametrization in Eq. (1), one can immediately obtain the maximal atmospheric mixing angle $\sin^2 \theta_{23} = 1/2$, a maximal CP-violating phase $\delta = 90^\circ$, and

$$\sin^2 \theta_{12} = \frac{2+\sqrt{3}}{10+\sqrt{3}} \approx 0.318, \quad \sin^2 \theta_{13} = \frac{2-\sqrt{3}}{12} \approx 0.022. \quad (7)$$

It is straightforward to observe that the predictions for θ_{12} and θ_{23} are very close to their global-fit values in Eqs. (3) and (4), while the prediction for θ_{13} is in perfect agreement with the Daya Bay result in Eq. (5). The Jarlskog invariant, which measures the magnitude of leptonic CP violation, turns out to be $\mathcal{J} = \sqrt{6}/72 \approx 3.4\%$. The next important step in neutrino oscillation experiments is to probe leptonic CP violation at the percent level.

The remaining part of this paper is organized as follows. In Sec. 2, we reconstruct the neutrino mass matrix M_ν by assuming the mixing pattern in Eq. (6), and point out a generalized μ - τ symmetry in the neutrino sector. In Sec. 3, we furthermore illustrate how to obtain such a mixing pattern in a class of seesaw models. Finally, we summarize our conclusions in Sec. 4.

2 Generalized μ - τ Symmetry

The lepton flavor mixing arises from the mismatch in the diagonalizations of charged-lepton and neutrino mass matrices. Once the mixing matrix is determined from the neutrino oscillation experiments, we can reconstruct the lepton mass matrices and learn something about the underlying symmetry structure in the lepton sector. However, it is obvious that such a reconstruction is not unique and depends on the flavor basis we have chosen. In the basis where the flavor eigenstates of charged leptons coincide with their mass eigenstates, the neutrino mass matrix can be reconstructed from neutrino mass eigenvalues and the MNSP matrix. Taking the TBM pattern for example, we have

$$M_\nu^0 = V_0 \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V_0^T \equiv \begin{pmatrix} a & b & b \\ b & c & d \\ b & d & c \end{pmatrix}, \quad (8)$$

where the four real parameters (a, b, c, d) are linear combinations of neutrino masses and satisfy the sum rule $a + b = c + d$. It is easy to verify that there exists a μ - τ exchange symmetry, which can be defined as $\nu_{eL} \rightleftharpoons \nu_{eL}$ and $\nu_{\mu L} \rightleftharpoons \nu_{\tau L}$ such that the neutrino mass term is invariant under these transformations [15]. The μ - τ symmetry is responsible for the bimaximal mixing with $\sin^2 \theta_{23} = 1/2$ and $\theta_{13} = 0$, while an extra Z_2 symmetry is needed to guarantee the condition $a + b = c + d$, leading to the trimaximal mixing with $\sin^2 \theta_{12} = 1/3$ [16]. Since the charged-lepton mass matrix in the chosen basis is diagonal, it should in general preserve a $U(1)^3$ symmetry. Various finite discrete symmetry groups, such as A_4 and S_4 , have been invoked to derive the TBM pattern at the leading order [5, 16].

Now that a significant deviation from the TBM pattern has been experimentally confirmed, we have to modify the TBM pattern, but in the most economical way. The basis idea is that the overall neutrino mass matrix can be decomposed into a symmetry-limit term and a perturbation term, and it is the latter that induces corrections to the TBM pattern and thus a nonvanishing θ_{13} . To account for both a nonzero θ_{13} and a maximal CP-violating phase, we take the corrections to V_0 as a rotation in the 2-3 complex plane:

$$V = V_0 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\vartheta & -is_\vartheta \\ 0 & -is_\vartheta & c_\vartheta \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}}c_\vartheta & -\frac{i}{\sqrt{3}}s_\vartheta \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}c_\vartheta - \frac{i}{\sqrt{2}}s_\vartheta & \frac{1}{\sqrt{2}}c_\vartheta - \frac{i}{\sqrt{3}}s_\vartheta \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}c_\vartheta + \frac{i}{\sqrt{2}}s_\vartheta & -\frac{1}{\sqrt{2}}c_\vartheta - \frac{i}{\sqrt{3}}s_\vartheta \end{pmatrix}, \quad (9)$$

where $s_\vartheta \equiv \sin \vartheta$ and $c_\vartheta \equiv \cos \vartheta$ with ϑ being a small rotation angle. If $\vartheta = 0$ is taken, then the TBM in Eq. (2) is reproduced. The above mixing pattern was first considered in Ref. [17] in the framework of a minimal type-I seesaw model [18], and later discussed in Ref. [19, 20] in light of the experimental evidence for a nonzero θ_{13} . Another simple but interesting case is to replace the rotation in 2-3 plane with the one in 1-3 plane, and its phenomenological implications have been studied in detail in Ref. [21, 22].

Compared with the standard parametrization in Eq. (1), such a mixing pattern predicts a maximal atmospheric mixing angle $\theta_{23} = 45^\circ$ and an intriguing correlation between the other two mixing angles [17]:

$$\sin^2 \theta_{12} = \frac{1}{3} (1 - 2 \tan^2 \theta_{13}) . \quad (10)$$

The salient feature of the relation in Eq. (10) is that $\theta_{12} \rightarrow 34^\circ$ as $\theta_{13} \rightarrow 9^\circ$, which are almost equal to their current best-fit values. Note that Eq. (10) has also been obtained in Ref. [23] from the constraints on the mixing matrix set by the structure of flavor symmetry group. As an explicit example, we set $\vartheta = \pi/12$ in Eq. (9) and then get the mixing pattern V' in Eq. (6). Its predictions have already been given in Eq. (7) and are in excellent agreement with current oscillation data. And the simple structure of V' is suggestive of some flavor symmetry.

Now we explore the implications of the MNSP matrix in Eq. (9) on the symmetry structure of neutrino mass matrix. Given the mixing matrix V and the neutrino masses m_i , one can immediately reconstruct the neutrino mass matrix:

$$M_\nu = \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \Delta M_\nu , \quad (11)$$

where ΔM_ν denotes the difference between the neutrino mass matrix M_ν and the counterpart M_ν^0 in the TBM case. More explicitly, we have

$$\Delta M_\nu = -\frac{m_2 + m_3}{6} \sin^2 \vartheta \begin{pmatrix} 2 & 2 & 2 \\ 2 & 5 & -1 \\ 2 & -1 & 5 \end{pmatrix} - i \frac{m_2 + m_3}{2\sqrt{6}} \sin 2\vartheta \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix} . \quad (12)$$

Note that ΔM_ν is proportional to $\sin \vartheta$, and it will vanish when $\vartheta = 0$, as expected. One can also easily figure out the neutrino mass matrix for the special case with $\vartheta = \pi/12$, corresponding to the mixing matrix V' in Eq. (6). However, we shall focus on the general case as in Eq. (11). It is straightforward to observe that the overall neutrino mass matrix takes the following form

$$M_\nu = \begin{pmatrix} x & y & y^* \\ y & z & w \\ y^* & w & z^* \end{pmatrix} , \quad (13)$$

where (x, w) are real while (y, z) are complex. In addition, two constraint relations $x + \text{Re}[y] = w + \text{Re}[z]$ and $2\text{Im}[y] = \text{Im}[z]$ should be satisfied. The direct diagonalization of M_ν in Eq. (13) leads to

$$\tan 2\vartheta = -\frac{2\sqrt{6}\text{Im}[y]}{\text{Re}[y] + 2\text{Re}[z]} . \quad (14)$$

Once ϑ is fixed, the mixing angles are determined by $\sin^2 \theta_{13} = \sin^2 \vartheta/3$ and Eq. (10). The remaining mixing parameters $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ arise from the intrinsic structure of M_ν and have nothing to do with the detailed values of the model parameters. More general discussions about M_ν in Eq. (13) without the constraint conditions can be found in Ref. [24].

We proceed to point out that a generalized μ - τ symmetry exists in the neutrino sector, which gives rise to the main features of M_ν in Eq. (13). In the chosen flavor basis, the neutrino mass term can be explicitly written as

$$\mathcal{L}_\nu = -\frac{1}{2} \sum_{\alpha,\beta} (M_\nu)_{\alpha\beta} \overline{\nu_{\alpha L}} \nu_{\beta L}^c + \text{h.c.} , \quad (15)$$

where $(M_\nu)_{\alpha\beta}$ (for $\alpha, \beta = e, \mu, \tau$) stand for the matrix elements of M_ν . If we define the generalized μ - τ symmetry by

$$\nu_{eL} \rightleftharpoons \nu_{eL}^c , \quad \nu_{\mu L} \rightleftharpoons \nu_{\tau L}^c , \quad \nu_{\tau L} \rightleftharpoons \nu_{\mu L}^c , \quad (16)$$

and require the neutrino mass term to be invariant under the above transformations, then the matrix elements $(M_\nu)_{\alpha\beta}$ are found to fulfill the following conditions:

$$(M_\nu)_{ee} = (M_\nu)_{ee}^* , \quad (M_\nu)_{\mu\tau} = (M_\nu)_{\mu\tau}^* , \quad (M_\nu)_{e\mu} = (M_\nu)_{e\tau}^* , \quad (M_\nu)_{\mu\mu} = (M_\nu)_{\tau\tau}^* . \quad (17)$$

These conditions are satisfied exactly by the neutrino mass matrix M_ν in Eq. (13). However, extra flavor symmetries or empirical assumptions have to be introduced to enforce the constraint relations among the four independent matrix elements.

3 Implications for Model Building

In the previous discussions, we have put aside the mechanism for neutrino mass generation. One of the simplest scenarios to accommodate tiny neutrino masses is to extend the standard model by three right-handed neutrinos N_{iR} for $i = 1, 2, 3$, which are singlets under the electroweak gauge group $\text{SU}(2)_L \times \text{U}(1)_Y$. In this scenario, the general lepton mass mass terms are

$$\mathcal{L}_\ell^I = -\overline{l}_L M_l E_R - \overline{\nu}_L M_D N_R - \frac{1}{2} \overline{N}_R^c M_R N_R + \text{h.c.} , \quad (18)$$

where M_l is the charged-lepton mass matrix, M_D and M_R are the Dirac and Majorana neutrino mass matrices, respectively. Note that M_R is the mass matrix for gauge singlets N_R and thus it is not subject to the electroweak gauge symmetry breaking at the scale $\Lambda_{\text{EW}} \sim 100$ GeV. For $\mathcal{O}(M_R) \gg \Lambda_{\text{EW}}$, the effective neutrino mass matrix is given by $M_\nu \approx -M_D M_R^{-1} M_D^T$, i.e., the type-I seesaw formula [25]. Therefore, the smallness of light neutrino masses is ascribed to the largeness of the masses of heavy Majorana neutrinos.

In the flavor basis where M_l and M_R are diagonal, i.e., $M_l = \text{Diag}\{m_e, m_\mu, m_\tau\}$ and $M_R = \text{Diag}\{M_1, M_2, M_3\}$, we apply the generalized μ - τ symmetry to the light neutrino fields ν_L as in the previous section. Furthermore, we assume the lepton mass terms are also invariant under $N_{iR} \rightleftharpoons N_{iR}^c$. In this case, the Dirac neutrino mass matrix turns out to be

$$M_D = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 e^{+i\varphi_1} & b_2 e^{+i\varphi_2} & b_3 e^{+i\varphi_3} \\ b_1 e^{-i\varphi_1} & b_2 e^{-i\varphi_2} & b_3 e^{-i\varphi_3} \end{pmatrix} , \quad (19)$$

where a_i , b_i and φ_i (for $i = 1, 2, 3$) are real parameters. One can verify that the effective neutrino matrix M_ν via the seesaw formula takes exactly the expected form in Eq. (13) with

$$x = \sum_{i=1}^3 \frac{a_i^2}{M_i}, \quad y = \sum_{i=1}^3 \frac{a_i b_i}{M_i} e^{i\varphi_i}, \quad w = \sum_{i=1}^3 \frac{b_i^2}{M_i}, \quad z = \sum_{i=1}^3 \frac{b_i^2}{M_i} e^{2i\varphi_i}. \quad (20)$$

If $a_i = b_i \cos \varphi_i$ holds for $i = 1, 2, 3$, the two constraint conditions $x + \text{Re}[y] = w + \text{Re}[z]$ and $\text{Im}[z] = 2\text{Im}[y]$ can be satisfied. Consequently, we obtain the mixing matrix in Eq. (9).

As another example, we now turn to the type-II seesaw model, where a $\text{SU}(2)_L$ triplet scalar Δ is introduced to generate a Majorana mass term for three light neutrinos [26]. In this model, the lepton mass terms are

$$\mathcal{L}_\ell^\text{II} = -\overline{l}_L M_l E_R - \frac{1}{2} \overline{\nu}_L M_\nu \nu_L^c + \text{h.c.}, \quad (21)$$

where $M_\nu = Y_\Delta \langle \Delta \rangle$ with Y_Δ of $\mathcal{O}(1)$ and $\langle \Delta \rangle \sim 0.1$ eV being the neutrino Yukawa coupling matrix and the vacuum expectation value of the neutral component of the triplet scalar, respectively. Therefore, the smallness of vacuum expectation value $\langle \Delta \rangle$ is responsible for the tiny neutrino masses.

In order to obtain the desired mixing pattern in Eq. (9), we follow a phenomenological approach and assume that the charged-lepton mass matrix M_l is real (i.e., $M_l = M_l^*$), and it furthermore satisfies two commutation relations:

$$[M_l, \mathcal{D}] = \mathbf{0}, \quad [M_l, \mathcal{P}_{\mu\tau}] = \mathbf{0}, \quad (22)$$

where the matrices \mathcal{D} and $\mathcal{P}_{\mu\tau}$ are defined as

$$\mathcal{D} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathcal{P}_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (23)$$

As observed in Ref. [21], the charged-lepton mass matrix M_l under these conditions can be exactly diagonalized by the TBM pattern, i.e., $V_0^T M_l V_0 = \text{Diag}\{m_e, m_\mu, m_\tau\}$. On the other hand, we impose a Z_2 symmetry on the neutrino fields, under which ν_{eL} is odd and $\nu_{\alpha L}$ (for $\alpha = \mu, \tau$) are even. Or equivalently, the neutrino mass matrix M_ν fulfills the commutation relation $[M_\nu, \mathcal{G}_\nu] = \mathbf{0}$ with

$$\mathcal{G}_\nu = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

Hence the neutrino mass matrix is block diagonal

$$M_\nu = \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & D \end{pmatrix}. \quad (25)$$

Without loss of generality, one can make A , B and D real by redefining the phases of neutrino fields. Such a redefinition will contribute to the Majorana-type CP-violating phases of the

MNSP matrix, which are not of our interest here. Hence only the parameter C is complex, and the Dirac CP-violating phase is in general arbitrary. If C is assumed to be imaginary (i.e., $C = i\tilde{C}$ with \tilde{C} real), the neutrino matrix M_ν in Eq. (25) can be diagonalized as follows

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\vartheta & -is_\vartheta \\ 0 & -is_\vartheta & c_\vartheta \end{pmatrix}^\dagger \begin{pmatrix} A & 0 & 0 \\ 0 & B & i\tilde{C} \\ 0 & i\tilde{C} & D \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\vartheta & -is_\vartheta \\ 0 & -is_\vartheta & c_\vartheta \end{pmatrix}^* = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad (26)$$

where the rotation angle is given by $\tan 2\vartheta = -2\tilde{C}/(B + D)$, while the neutrino masses are

$$\begin{aligned} m_1 &= A, \\ m_2 &= \frac{1}{2} \left[\sqrt{(D + B)^2 + 4\tilde{C}^2} - (D - B) \right], \\ m_3 &= \frac{1}{2} \left[\sqrt{(D + B)^2 + 4\tilde{C}^2} + (D - B) \right]. \end{aligned} \quad (27)$$

As we have already seen, the mixing pattern in Eq. (9) with $\vartheta = \pi/12$ is well consistent with current neutrino oscillation data. Therefore, the relation $B + D + 2\sqrt{3}\tilde{C} = 0$ holds in this case. Such a relation, together with the absolute neutrino mass m_1 and two independent neutrino mass-squared differences $(\Delta m_{21}^2, \Delta m_{31}^2)$, uniquely determines the model parameters.

4 Summary

Motivated by the recent experimental evidence for a relatively large θ_{13} , we suggest a novel neutrino mixing pattern that predicts $\sin^2 \theta_{12} = (2 + \sqrt{3})/(10 + \sqrt{3}) \approx 0.318$, $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{13} = (2 - \sqrt{3})/12 \approx 0.022$ and a maximal CP-violating phase $\delta = 90^\circ$. These predictions are very close to the best-fit values of $\theta_{12} \approx 34^\circ$, $\theta_{23} \approx 45^\circ$ and $\theta_{13} \approx 9^\circ$. The leptonic CP violation, characterized by the Jarlskog invariant $\mathcal{J} = \sqrt{6}/72 \approx 3.4\%$, is expected to be measured in the future long-baseline neutrino oscillation experiments. Such a mixing pattern can be viewed as the minimal modification of the well-known tri-bimaximal mixing pattern through the rotation in 2-3 complex plane with a small rotation angle $\vartheta = \pi/12$. If one identifies ϑ with the Cabibbo angle $\theta_C \approx 13^\circ$, the resultant MNSP matrix should also be compatible with current neutrino oscillation experiments. In this case, it might be possible to relate the flavor mixing in the lepton sector to that in the quark sector.

In the general case with an arbitrary angle ϑ , we reconstruct the neutrino mass matrix M_ν in the flavor basis where the charged-lepton mass matrix is diagonal. Furthermore, we point out that there exists a generalized μ - τ symmetry, defined by the transformations $\nu_{eL} \rightleftharpoons \nu_{eL}^c$ and $\nu_{\mu L} \rightleftharpoons \nu_{\tau L}^c$, in the neutrino sector. Two examples in the type-I and type-II seesaw models have been worked out to derive the desired neutrino mass matrix in a phenomenological way. We believe that this investigation should be helpful in understanding the lepton flavor mixing with a relatively large θ_{13} . In particular, the imposed symmetries in the charged-lepton and neutrino sectors in these two examples may be instructive for the model building based on some discrete flavor symmetries.

The ongoing and forthcoming neutrino oscillation experiments will provide us with more precise values of three neutrino mixing angles, so the predictions of the suggested mixing pattern are easily to be confirmed or refuted by the future oscillation data. The maximal CP violation at the percent level will be soon tested in the long-baseline oscillation experiments. In any case, the experimental hints on neutrino mixing parameters are definitely desirable and will finally guide us to the true theory of fermion masses, flavor mixing and CP violation.

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